

## UNIVERSITI TEKNOLOGI MALAYSIA

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**JUDUL: CONTROL OF A CART- BALL SYSTEM:COMPARISON BETWEEN  
MODEL BASED AND FUZZY LOGIC CONTROLLER**

**SESI PENGAJIAN: 2005/2006**

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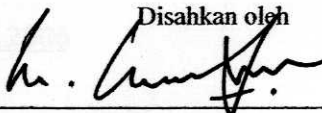
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**CONTROL OF A CART-BALL SYSTEM: COMPARISON BETWEEN MODEL  
BASED AND FUZZY LOGIC CONTROLLERS**

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A project report submitted in partial fulfilment of the  
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I declare that this thesis "*Control of A Cart-Ball System: Comparison Between Model Based And Fuzzy Logic Controllers*" is the result of my own research except for works that have been cited in the reference. The thesis has not been accepted any degree and not concurrently submitted in candidature of any other degree.

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To my dearest father, mother, family and Kolej Universiti Teknikal Kebangsaan  
Malaysia (KUTKM) for their encouragement and blessing  
To my lovely wife and son for their support and caring ... ..



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## **ABSTRACT**

A cart-ball system is a challenging system from the control engineering point of view. This is due to the nonlinearities, multivariable and non-minimum phase behavior presented in the system. This thesis is concerned with the problem of modeling and control of a cart-ball system such that to balance the ball on the top of the arc and at the same time to place the cart at a desired position. Two types of the controllers will be synthesized in order to control the system. One is the model based controller i.e. State-Feedback Controller and second is a Fuzzy Logic Controller. The first stage is to develop the mathematical model of a cart-ball system based on the state-space theory. Then, the linearization technique will be applied to the nonlinear model so that the design of the State-Feedback Controller can be accomplished. The second stage is to design the Fuzzy Logic Controller to be applied to the system. The final stage is to carry out the simulation work of both controllers for comparison purpose. The simulation work is done using a MATLAB/SIMULINK platform.



## ABSTRAK

Sistem kereta-bola merupakan sebuah sistem yang mencabar dari perspektif kejuruteraan kawalan. Ini kerana sistem ini memiliki sifat-sifat tidak linear, mempunyai banyak pembolehubah dan memiliki fasa yang tidak minimum. Tesis ini menumpukan kepada penghasilan model matematik dan juga merekabentuk sistem kawalan kepada sistem kereta-bola ini bagi memastikan keseimbangan bola di atas kereta dan pada masa yang sama memastikan kereta berhenti pada jarak yang ditetapkan. Dua jenis pengawal akan di sintesis, pertama ialah Pengawal Suap-Balik Keadaan dan kedua adalah Pengawal Fuzzy Logik. Peringkat pertama yang dilaksanakan adalah membangunkan model matematik untuk sistem ini berdasarkan kepada teori ruang-keadaan. Selepas itu, teknik penglinearan akan dijalankan ke atas model tidak linear ini untuk membolehkan Pengawal Suap-Balik Keadaan di rekabentuk. Peringkat kedua adalah merekabentuk Pengawal Fuzzy Logik untuk mengawal sistem kereta-bola. Simulasi kepada kedua-dua pengawal akan dijalankan pada peringkat akhir untuk tujuan perbandingan. Segala kerja simulasi dijalankan dengan menggunakan perisian MATLAB/SIMULINK.

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## LIST OF ABBREVIATIONS

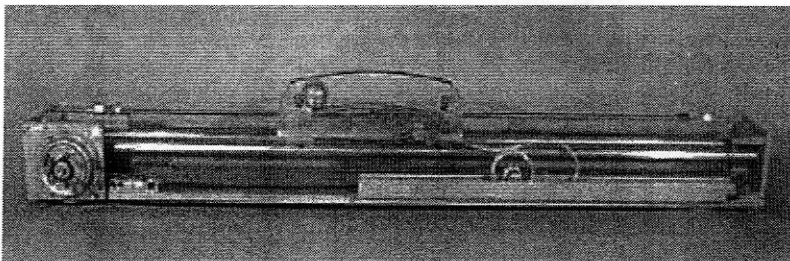
SISO	Single Input Single Output
MIMO	Multiple Input Multiple Output
SFC	State Feedback Controller
FLC	Fuzzy Logic Controller

## CHAPTER 1

### INTRODUCTION

#### .1 Overview

A cart-ball system is basically an inverted pendulum problem, which is a much used as a benchmark problem. The control objectives are to balance the ball on the top of the arc and at the same time place the cart at the desired position. So, an extremely good control strategy is needed in order to achieve the objective target. Example of a laboratory cart-ball system is as shown in Figure 1.1 below.



**Figure 1.1** Example of a laboratory cart-ball system [Jantzen, 1999]

By pushing the cart left and right manually, it is possible to get the ball on the top of the arc, but it is impossible to position the cart at the particular position at the same time. An automatic control system can do that with a good control strategy. In order to control the cart-ball system the cart position and the ball angle from vertical are measured variables, and manipulated variable is the horizontal force acting on the cart.

A cart-ball system can demonstrate some basic concepts in control being nonlinear, non-minimum phase and multivariable. So it can teach electrical engineers about automatic control. The laboratory equipment of a cart-ball system already built by the Janzten (1999) and the mathematical modeling is published by Jorgensen (1974). However, both of the papers do not consider any disturbances in their modeling..

This project will study the cart-ball system with the disturbance (horizontal force applied to the ball). The effect of the disturbance to the system will be studied in order to design a good controller. A good controller must be designed such that it can compensate the existence of the disturbance to the system and can control the system well. Thus, with the existence of the disturbance will make the controller design is tougher.

## 1.2 Objectives

The objectives of this research are as follows:

- a. To determine the state-space representation of a cart-ball system with the disturbance.
- b. To synthesis the model based controller i.e state feedback controller to control the system.
- c. To synthesis the fuzzy logic controller to control the system.
- d. To carry out the simulation works of both controllers when applied to the system for comparison purpose.

## .3 Scope of Works

he scopes of work for this project are

- a. A cart-ball system as described in Jantzen (1999).
- b. Application of the model based controllers (state feedback controller) and fuzzy logic controller in order to balance the ball on the top of the arc and at the same time place the cart at a desired position.
- c. The comparison between all of these controllers when applied to a cart-ball system will be studied i.e transient response and steady-state error.
- d. Simulation work will be performed under the MATLAB/SIMULINK platform.



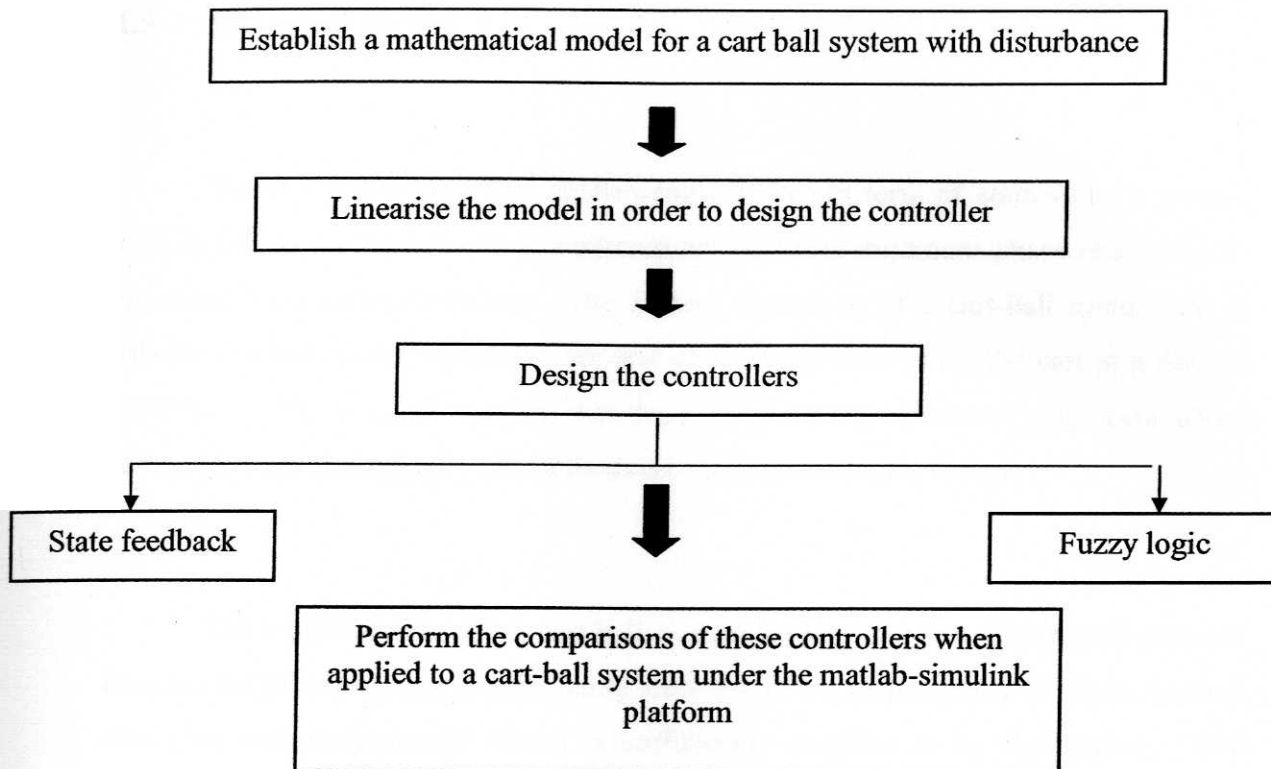
## 1.4 Research Methodology

Figure 1.2 shows the block diagram of the methodology taken in order to accomplish the task. It can be seen that firstly the mathematical model of a cart ball system must be derived. The mathematical model is based on the state space theory. Mathematical modeling is needed in order to design the controller and to get the equation for the plant (cart-ball). The plant equation must be as close as the actual plant (nonlinear).

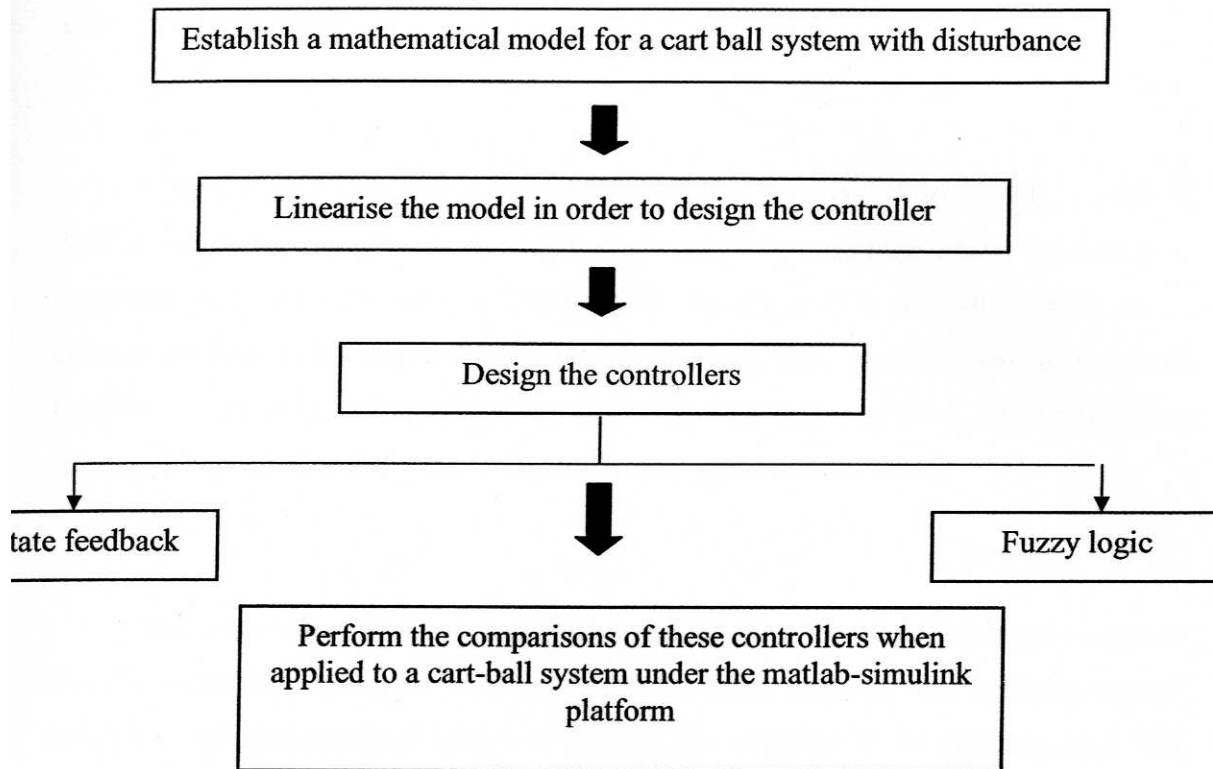
After mathematical model of the system is established then the equations must be linearised around the origin as to design the controller because the State-Feedback Controller only deals with the linear equations.

The performance of the model based controller and fuzzy logic controller will be studied and comparison between these controllers will be performed.

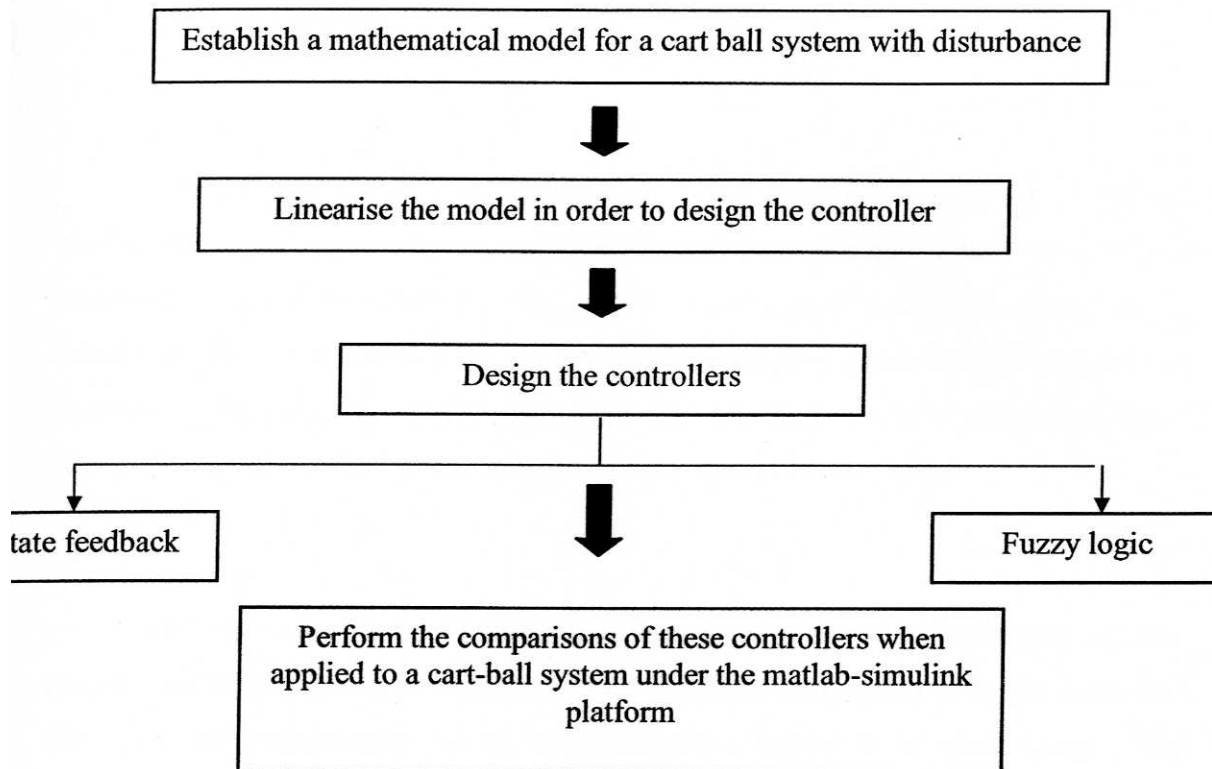




**Figure 1.2** Chart shows the methodology of the research



**Figure 1.2** Chart shows the methodology of the research



**Figure 1.2** Chart shows the methodology of the research

## 1.5 Literature Review

The cart-ball system is a challenging problem in term of controlling a system. This is due to the nonlinearities, multivariable and non-minimum phase characteristic presented by a cart-ball system. The control objectives of a cart-ball system are to balance the ball on the top of the arc and at the same time place the cart at a desired position. The cart ball system was built for teaching electrical engineers about automatic control, originally with a focus on state-space control theory.

The laboratory rig for a cart-ball system is done for the educational purpose because the laboratory rig is sufficiently slow for visual inspection of different control strategies and mathematical model is sufficiently complex to be challenging. The approach is to develop the mathematical model from *first principles*, i.e., the basic laws of physics. After that the linearization was applied to the model in order to make it easier to discuss possible controller configurations [Jantzen, 1999].

Many researches were carried out researches to control an inverted pendulum system. Various control strategies have been proposed by numerous researchers for controlling the inverted pendulum such that the system is stable as well as the cart is move to the desired position. The approaches varied from the classical control to the advanced control. PID controller was design to control the inverted pendulum problem [Jantzen, 1999]. The drawback of the PID controller is it only can control for a Single-Input-Single-Output (SISO) system. It means that the PID controller only can control either for the position of the cart or angle of the ball at a one time [Jantzen, 1999]